

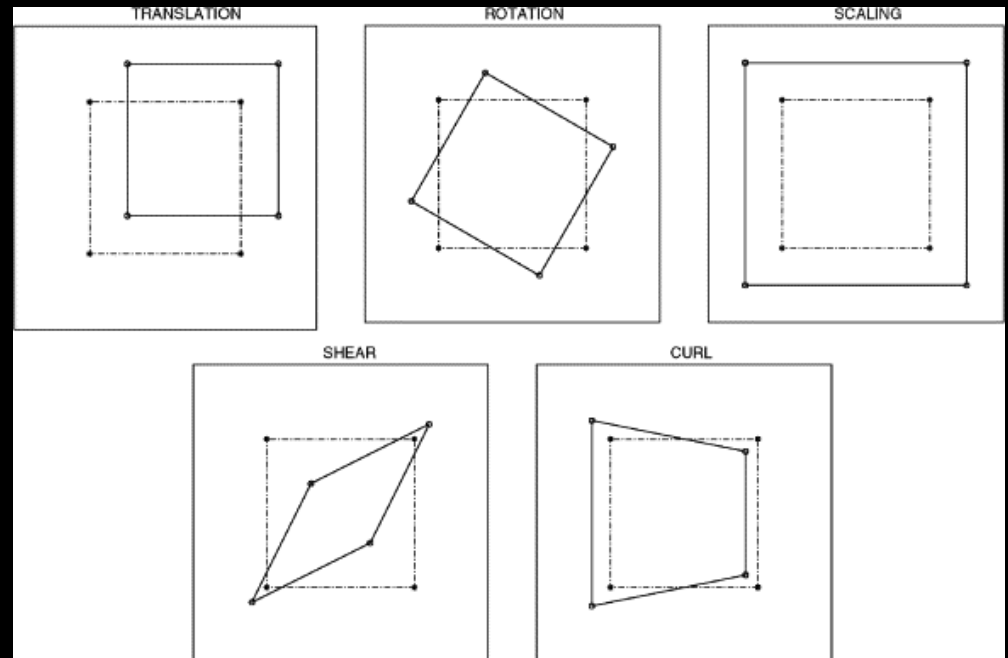


# Računarska grafika

predavanja  
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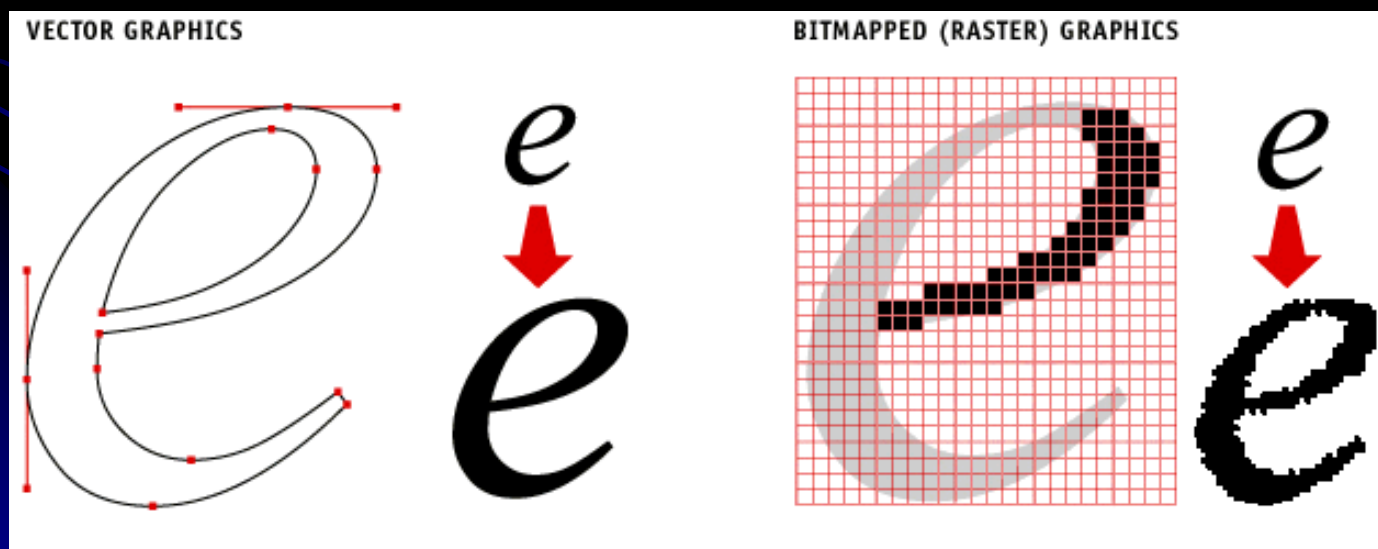
# Geometrijske transformacije

- Vektorska grafika
- Homogene koordinate
- 2D Primitivi
- 2D translacija
- 2D rotacija
- 2D skaliranje
- Inverzne transformacije



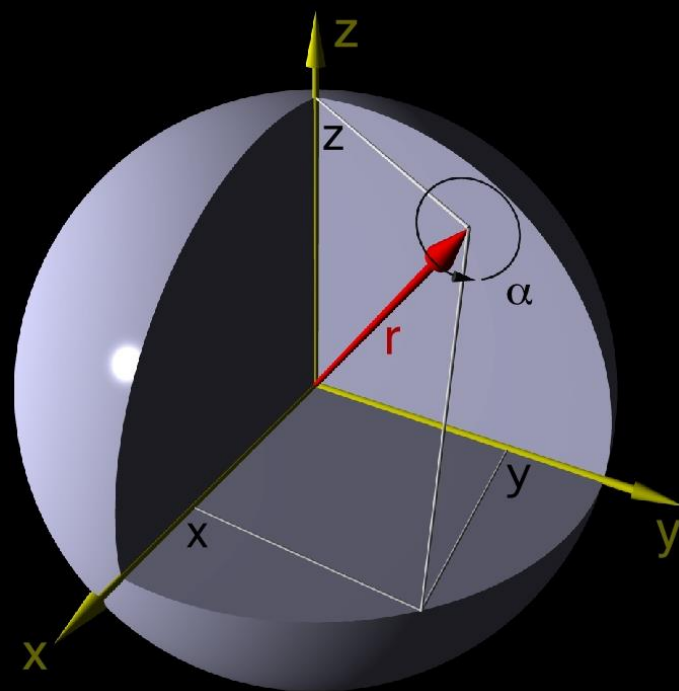
# Vektorska grafika

- U rasterskoj grafici slika je predstavljena mrežom piksela.
- Nedostaci: zahtijeva puno memorije, povećanjem slike gubi se kvalitet.



# Vektorska grafika

- Vektorska grafika sliku opisuje geometrijskim likovima: tačke, linije, krugovi, funkcije,...
- Položaj geometrijskih likova određuje se koordinatama u izabranom koordinatnom sistemu (2D:  $x, y$  ili  $r, \theta$ ; 3D:  $x, y, z$  ili  $r, \theta, z$ ).



# Homogene koordinate

- Uobičajene transformacije u računarskoj grafici: translacija, rotacija, skaliranje, projekcije, se lakše izvode ako se koriste homogene koordinate, jer se onda te transformacije mogu implementirati u obliku operacija s matricama.
- Homogene koordinate je uveo Möbius 1827.
- Prednost je u tome što se sve tačke mogu prikazati konačnim brojevima (čak i  $\infty$ )

# Homogene koordinate

- Prikaz tačke parom brojeva  $(x,y)$  se zamjenjuje prikazom sa tri tačke  $(x,y,h)$
- Skalarni proizvod homogene koordinate daje istu tačku:  $(2,3,5) = (4,6,10)$
- To znači da se ista tačka može predstaviti sa više različitih homogenih koordinata.
- Bar jedna homogena koordinata mora biti različita od nule;  $(0,0,0)$  nije dozvoljena
- $h=0$  daje tačke "u beskonačnosti"

# 2D Primitivi

- 2D tačka:  $V(x, y) \rightarrow X = (x', y', h)$  ili  $X = (x_1, x_2, x_3)$

$$x = \frac{x_1}{x_3} \quad y = \frac{x_2}{x_3}$$

- 2D pravac:

- implicitni oblik:

$$a \cdot x + b \cdot y + c = 0$$

- uvođenje homogene koordinate:

$$a \cdot \frac{x_1}{x_3} + b \cdot \frac{x_2}{x_3} + c = 0$$

- homogena jednačina:

$$a \cdot x_1 + b \cdot x_2 + c \cdot x_3 = 0$$

# 2D Primitivi

- 2D pravac:

- matrični

zapis:

$$X \cdot G = a \cdot x_1 + b \cdot x_2 + c \cdot x_3 = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

- vektor normale:

$$n = \begin{bmatrix} a & b \end{bmatrix}$$

- vektor tangente:

$$t = \begin{bmatrix} b & -a \end{bmatrix}$$

- dogovor:

$$X \cdot G \begin{cases} > 0, & X \text{ je "iznad" pravca} \\ = 0, & X \text{ je na pravcu} \\ < 0, & X \text{ je "ispod" pravca} \end{cases}$$

# 2D primitivi

- Dvije tačke određuju pravac (vektorski proizvod)

$$X_1 = (x_1 \ y_1 \ h_1)$$

$$X_2 = (x_2 \ y_2 \ h_2)$$

$$G = X_1 \times X_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & h_1 \\ x_2 & y_2 & h_2 \end{vmatrix} =$$

$$= \left( \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \end{bmatrix} \cdot \begin{bmatrix} b_1 c_2 - b_2 c_1 \\ -(a_1 c_2 - a_2 c_1) \\ a_1 b_2 - a_2 b_1 \end{bmatrix} \right)^T = \begin{bmatrix} x \\ y \\ h \end{bmatrix}^T = X^T$$

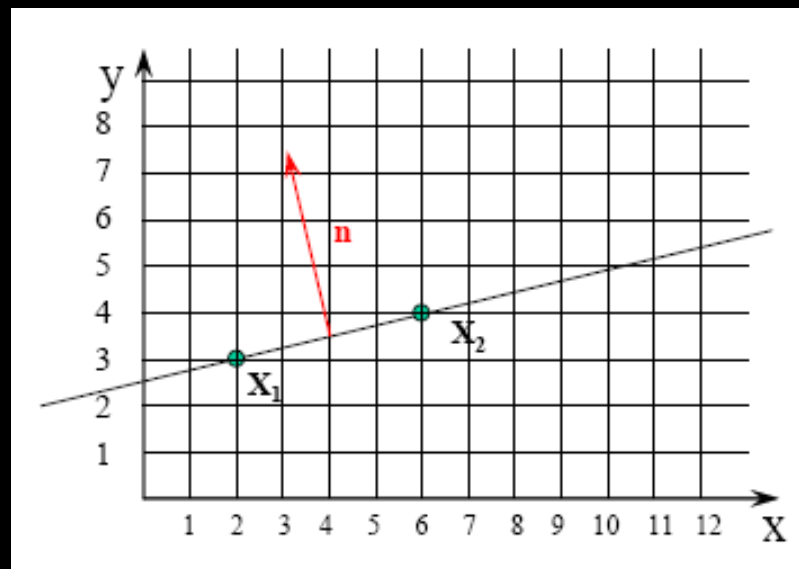
# Primjer

- $X_1=(2 \ 3 \ 1), \quad X_2=(6 \ 4 \ 1)$

$$G = X_1 \times X_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 1 \\ 6 & 4 & 1 \end{vmatrix}$$

$$G = \left( \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \end{bmatrix} \cdot \begin{bmatrix} 3-4 \\ -(2-6) \\ 8-18 \end{bmatrix} \right)^T = \left( \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 4 \\ -10 \end{bmatrix} \right)^T$$

$$G = -x_1 + 4x_2 - 10x_3 = -x + 4y - 10 = 0$$

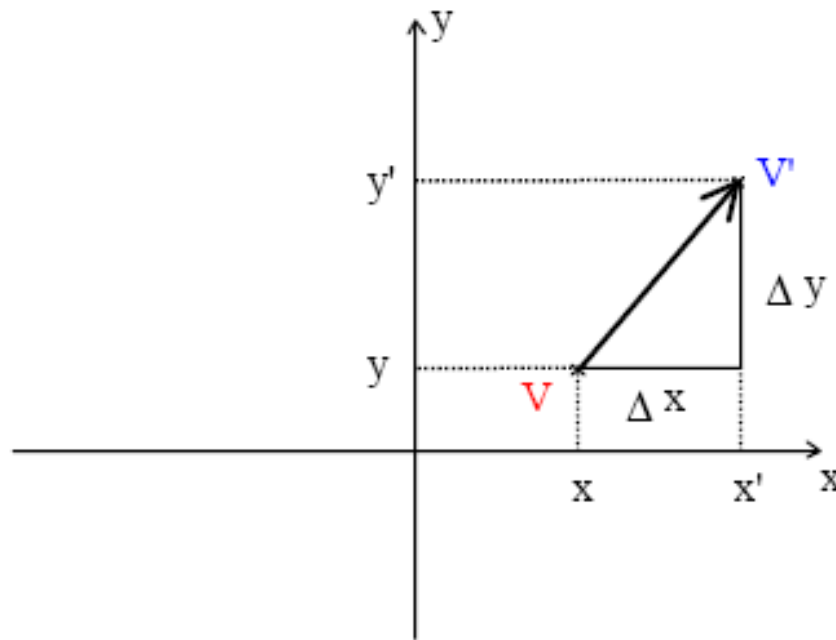


# 2D translacija

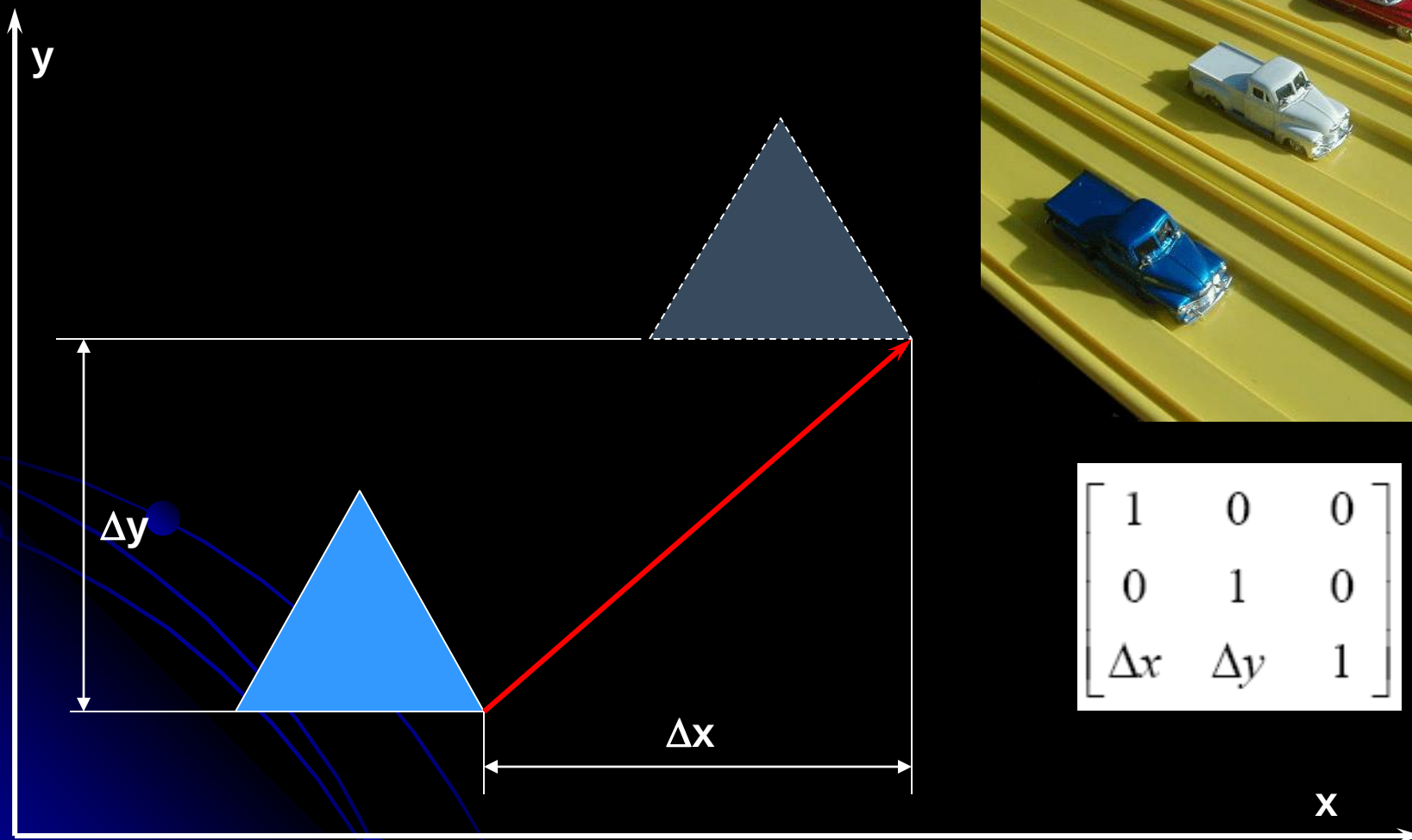
$$\begin{aligned}x' &= x + \Delta x \\ y' &= y + \Delta y\end{aligned}\quad \begin{bmatrix} x' & y' & h' \end{bmatrix} = \begin{bmatrix} x & y & h \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \Delta x & \Delta y & 1 \end{bmatrix}$$

$$\mathbf{V}' = \mathbf{V} \cdot \mathbf{T}$$

T- matrica translacija



# 2D translacija

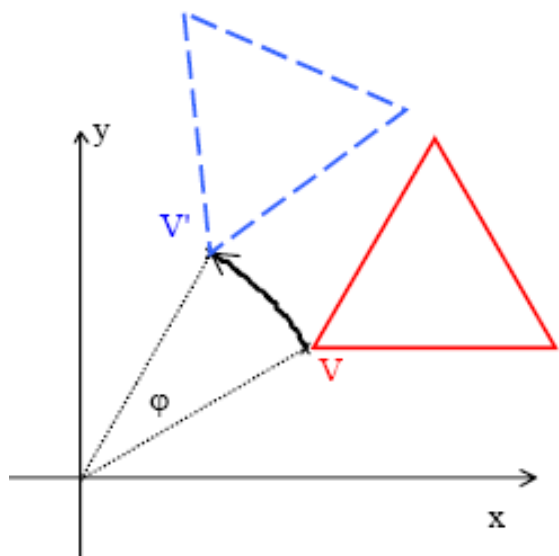


# 2D rotacija

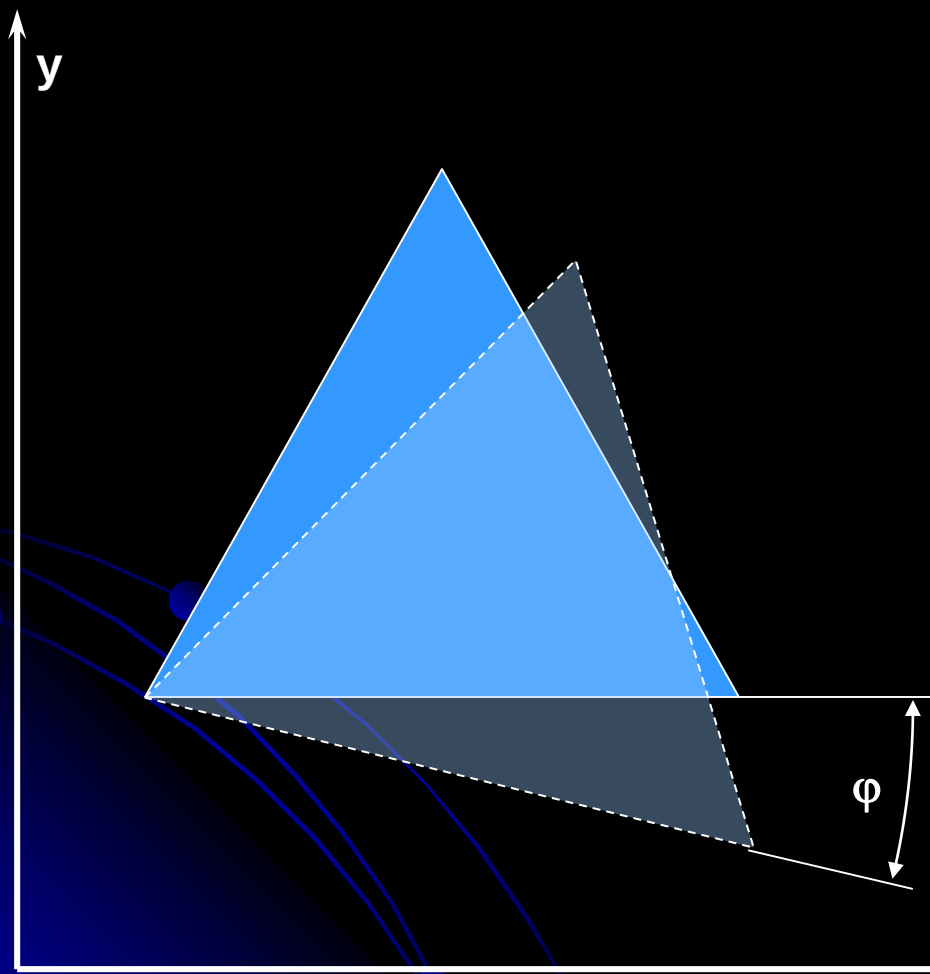
$$x' = x \cos \varphi - y \sin \varphi$$

$$y' = x \sin \varphi + y \cos \varphi$$

$$\begin{bmatrix} x' & y' & h' \end{bmatrix} = \begin{bmatrix} x & y & h \end{bmatrix} \cdot \begin{bmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{V}' = \mathbf{V} \cdot \mathbf{R}$$



# 2D rotacija



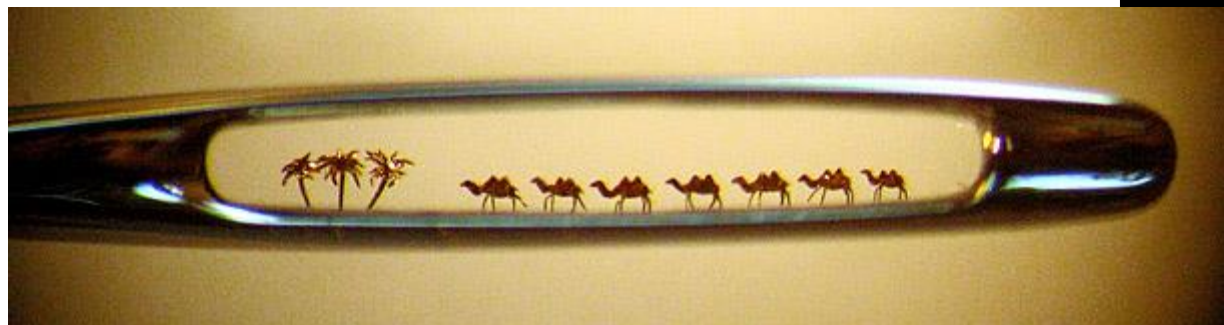
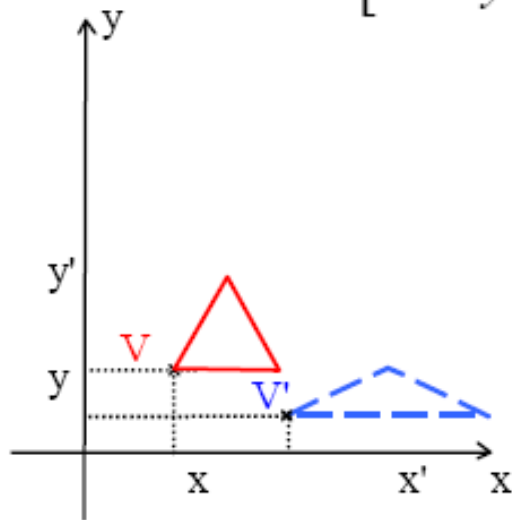
$$\begin{bmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

x

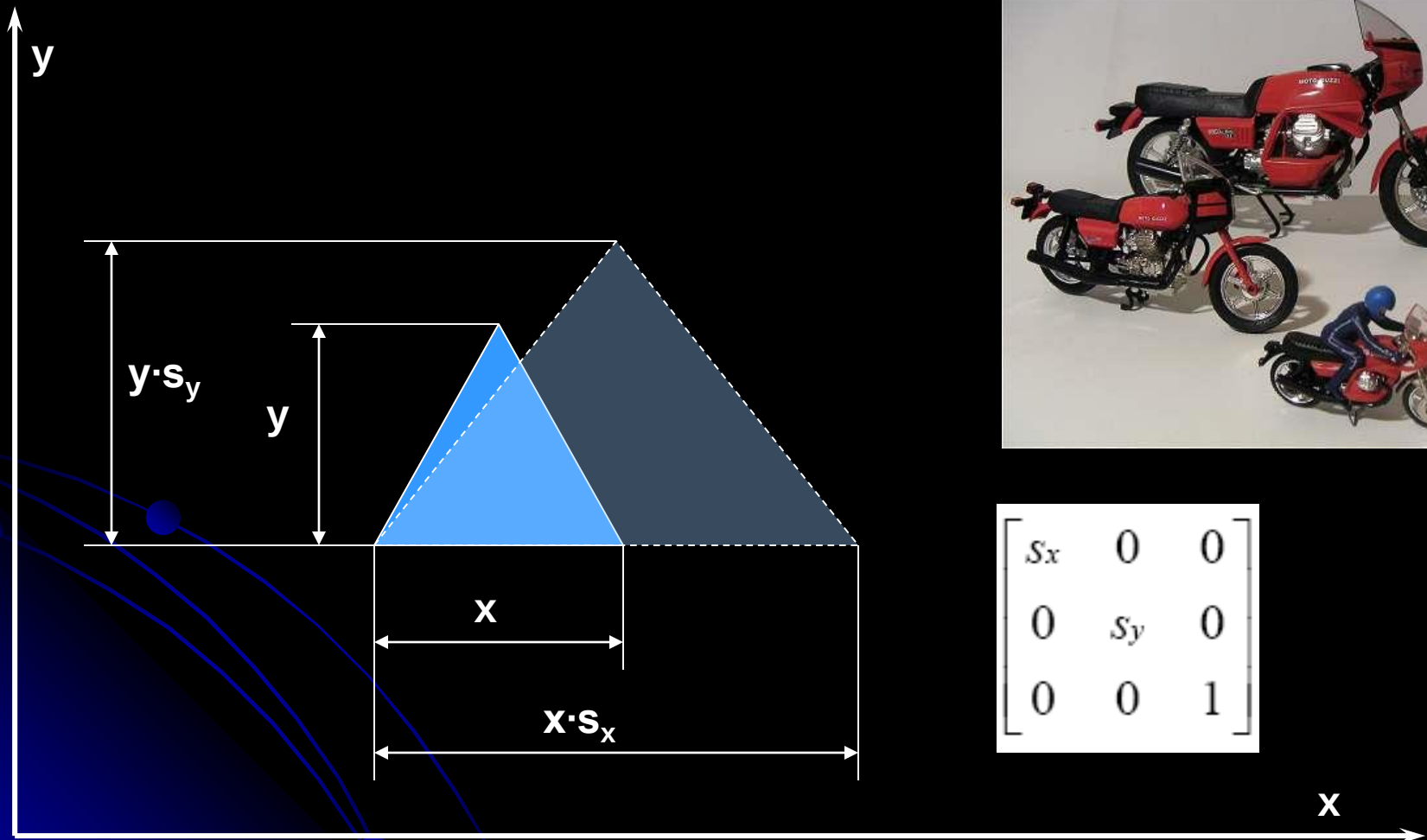
# 2D skaliranje

$$\begin{aligned} x' &= x S_x \\ y' &= y S_y \end{aligned} \quad \begin{bmatrix} x' & y' & h' \end{bmatrix} = \begin{bmatrix} x & y & h \end{bmatrix} \cdot \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{V}' = \mathbf{V} \cdot \mathbf{S}$$

$$\begin{bmatrix} x' & y' & h' \end{bmatrix} = \begin{bmatrix} x & y & h \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



# 2D skaliranje



$$\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

# Inverzne transformacije

- Inverzna matrica translacije daje pomak u suprotnom smjeru

$$\mathbf{T}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \Delta x & \Delta y & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\Delta x & -\Delta y & 1 \end{bmatrix}$$

- Inverzna matrica rotacije daje rotaciju u suprotnom smjeru

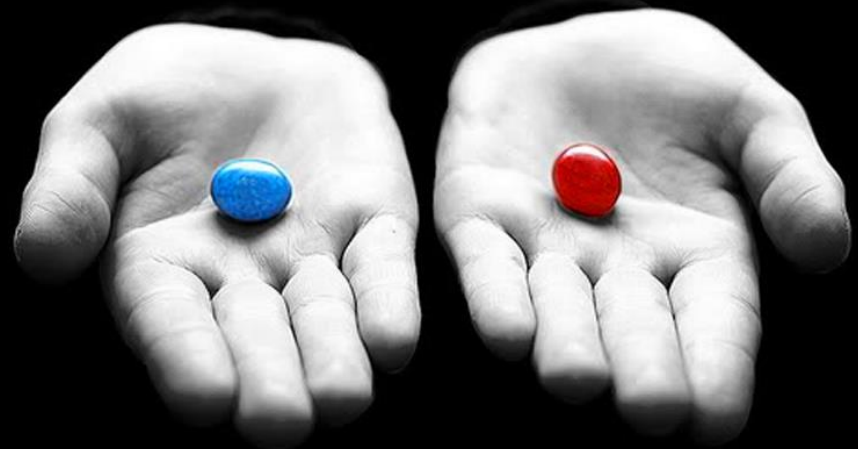
$$\mathbf{R}^{-1} = \begin{bmatrix} \cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \cos(-\varphi) & \sin(-\varphi) & 0 \\ -\sin(-\varphi) & \cos(-\varphi) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos \varphi & -\sin \varphi & 0 \\ \sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

# Inverzne transformacije

- Inverzna matrica skaliranja odgovara inverznoj transformaciji; povećanje odgovara smanjivanju

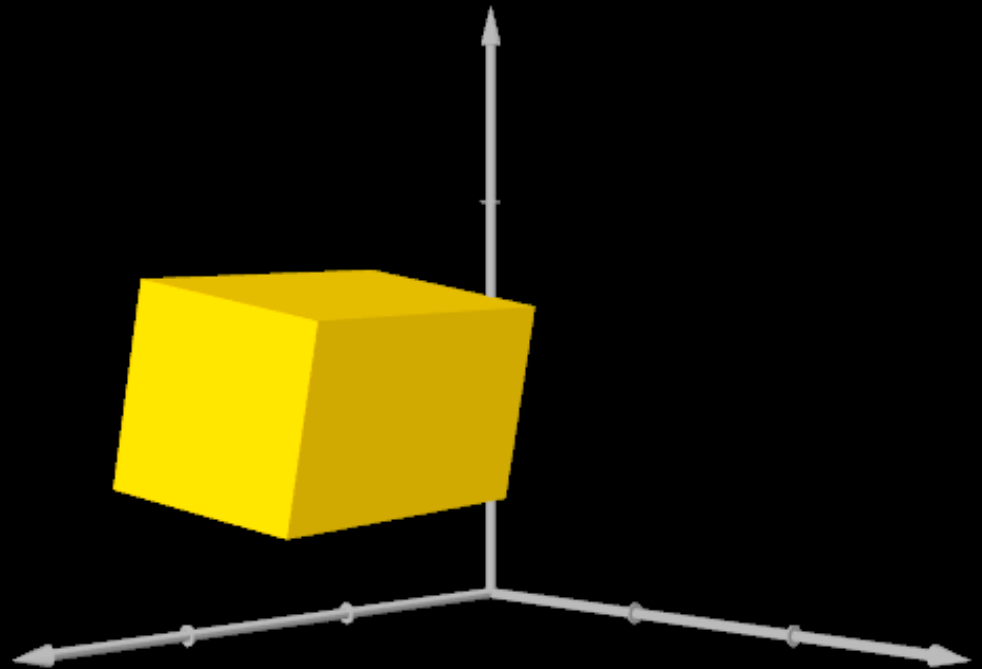


$$S^{-1} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{s_x} & 0 & 0 \\ 0 & \frac{1}{s_y} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



# Geometrijske transformacije

- Homogene koordinate
- 3D Primitivi
- Bulove operacije
- 2D-3D operacije
- 3D translacija
- 3D rotacija
- 3D skaliranje

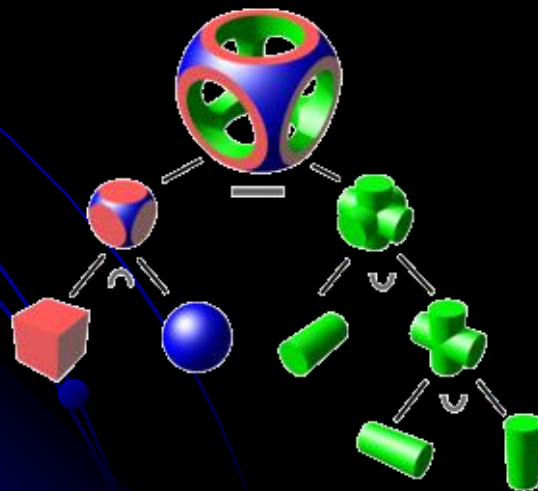


# Homogene koordinate

- Prikazu tačke sa tri koordinate  $(x,y,z)$  se dodaje homogena koordinata  $(x,y,z,h)$
- Homogenizacija koordinata je transformacija tačke  $(x,y,z,h)$  u  $(x/h,y/h,z/h,1)$
- Bar jedna homogena koordinata mora biti različita od nule;  $(0,0,0,0)$  nije dozvoljena
- $h=0$  daje tačke "u beskonačnosti"

# 3D Primitivi

- U 3D vektorskoj grafici, slika se sastoji od primitiva: osnovnih geometrijskih tijela koja kombinovanjem daju složenije oblike.
- Za kombinovanje se koriste operacije Bulove algebre.



# 3D Primitivi

- 3D tačka:  $V(x, y, z) \rightarrow X = (x', y', z', h)$  ili  $X = (x_1, x_2, x_3, x_4)$

$$x = \frac{x_1}{x_4} \quad y = \frac{x_2}{x_4} \quad z = \frac{x_3}{x_4}$$

- 3D pravac:

- implicitni oblik:

$$a \cdot x + b \cdot y + c \cdot z + d = 0$$

- u homogenim koordinatama:

$$a \cdot \frac{x_1}{x_4} + b \cdot \frac{x_2}{x_4} + c \cdot \frac{x_3}{x_4} + d = 0$$

- homogena jednačina:

$$a \cdot x_1 + b \cdot x_2 + c \cdot x_3 + d \cdot x_4 = 0$$

# 3D Primitivi

- U matričnom obliku:

- 3D tačka:

$$\begin{bmatrix} x & y & z & 1 \end{bmatrix}^T = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- 3D pravac:

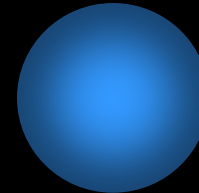
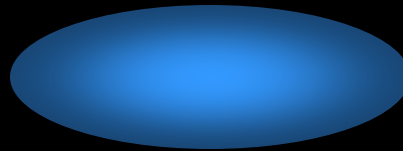
$$a \cdot x + b \cdot y + c \cdot z + d(\cdot 1) = \begin{bmatrix} x & y & z & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = 0$$

# 3D primitivi

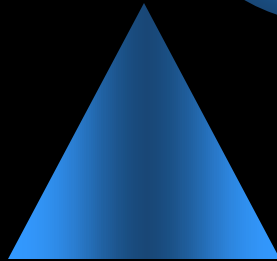
- Kvadar (kocka)



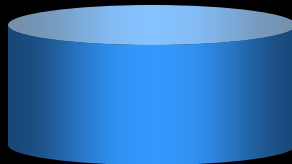
- Elipsoid (sfera)



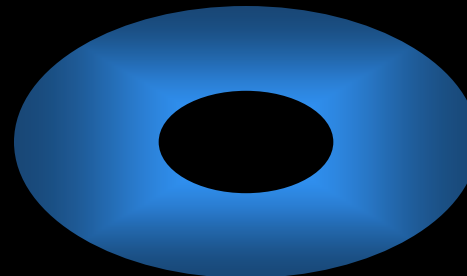
- Konus



- Cilindar

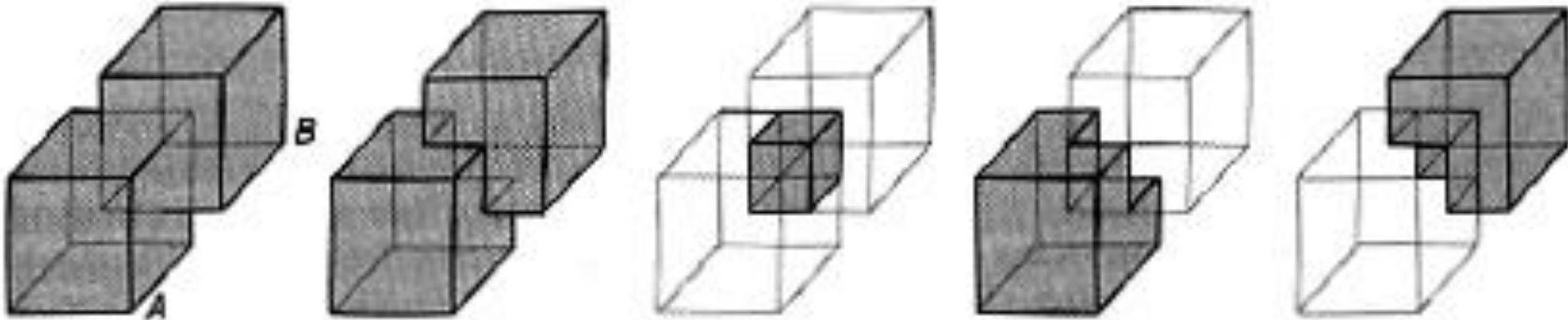


- Torus



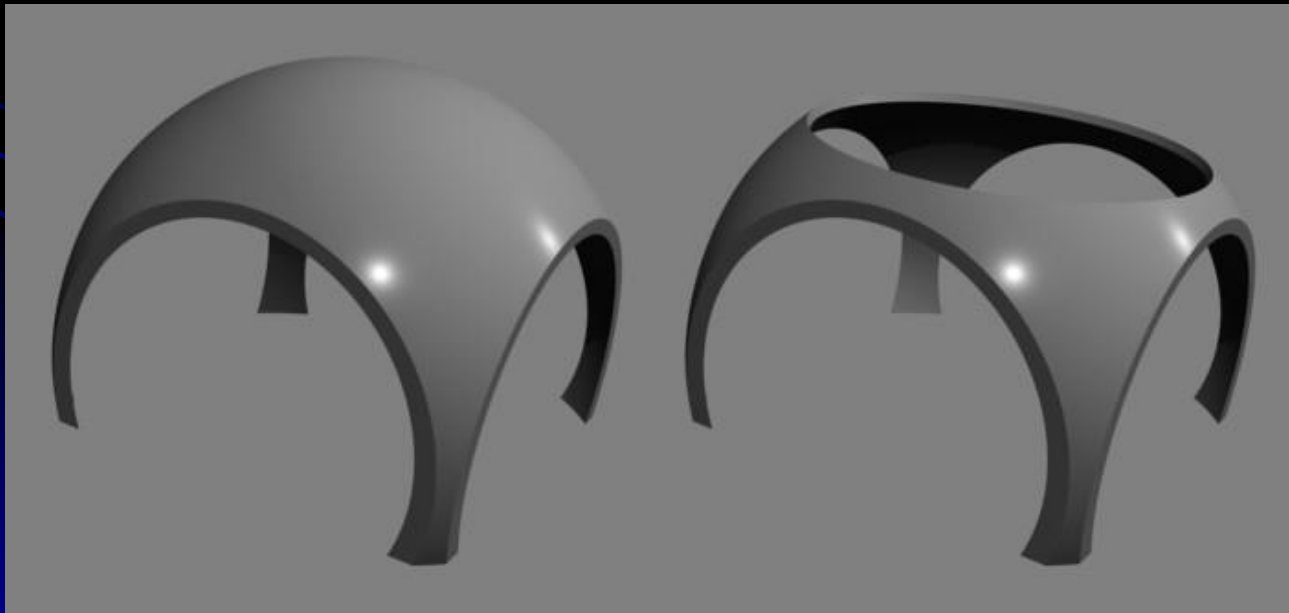
# Bulove operacije

- Kombinovanjem 3D primitiva dobiju se složeniji oblici
- Koriste se 3 operacije: unija, presjek i razlika



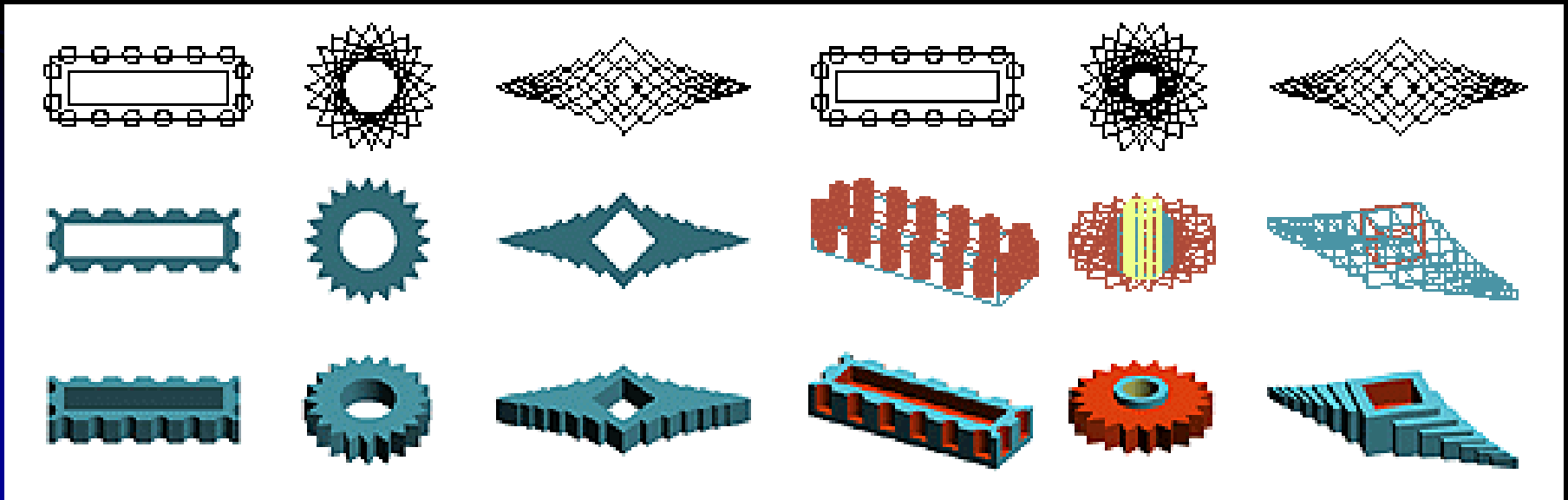
# Bulove operacije

- Zadatak:
  - a) Koji primitivi su potrebni da bi se napravio oblik kao na desnoj slici?
  - b) Koliko je potrebno operacija razlike?



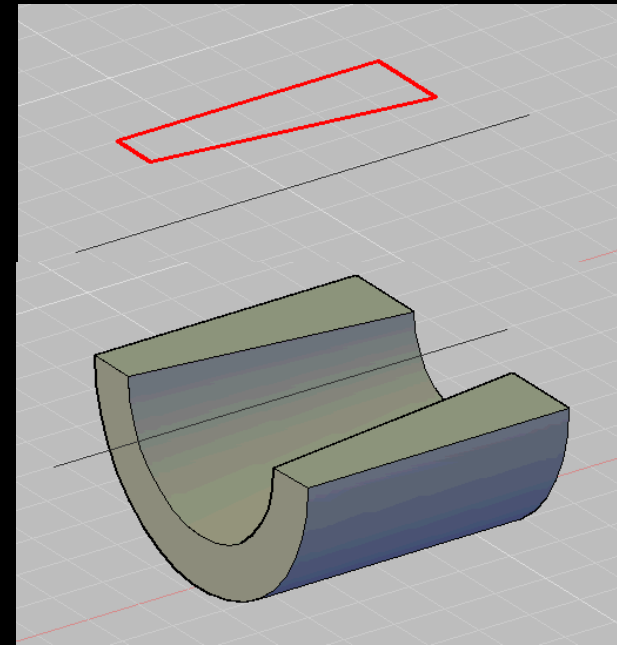
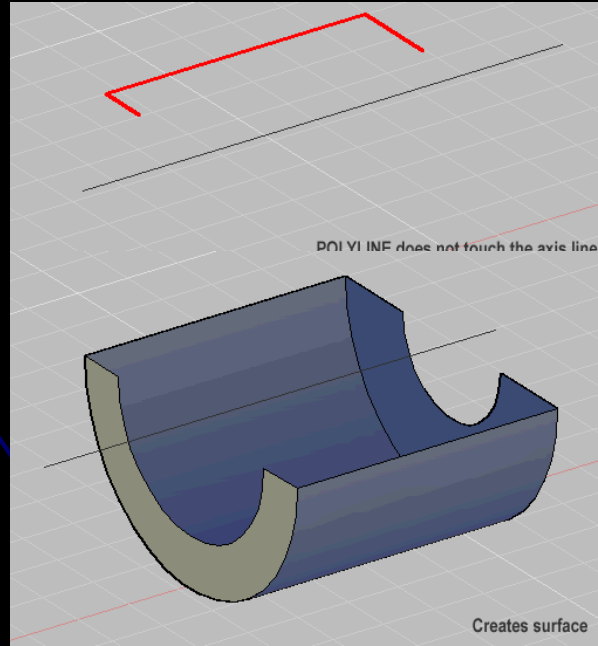
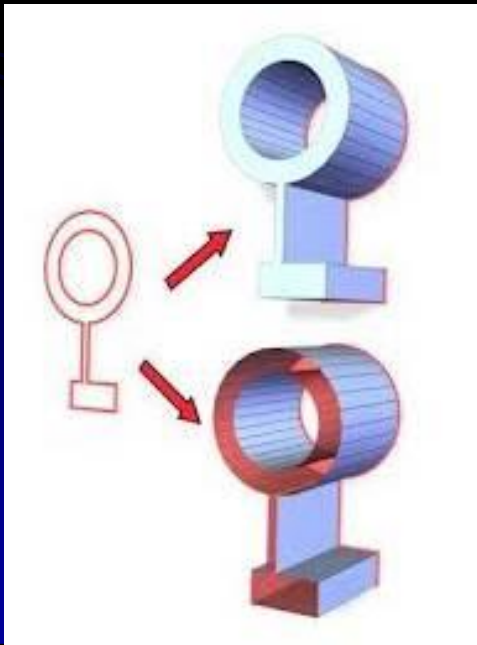
# Bulove operacije

- Bulove operacije se mogu vršiti i na 2D konturama i na 3D objektima.
- 2D konture se mogu nakon toga koristiti kao osnova za formiranje 3D geometrije.



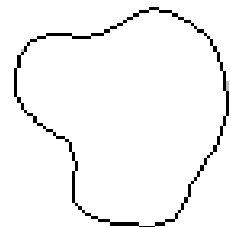
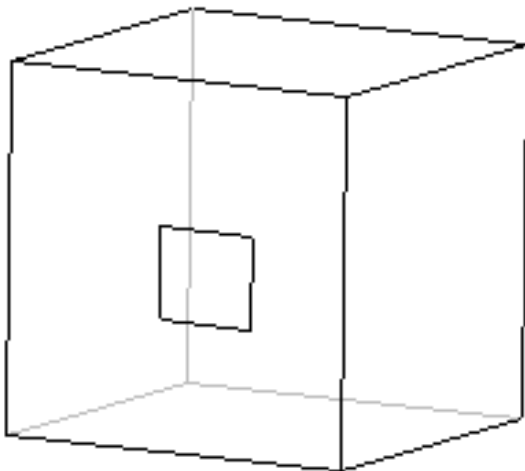
# 2D-3D operacije

- Neke geometrijske oblike nije moguće dobiti kombinovanjem 3D primitiva.
- Tada se koriste 2D-3D operacije koje od 2D kontura generišu prostorne oblike.



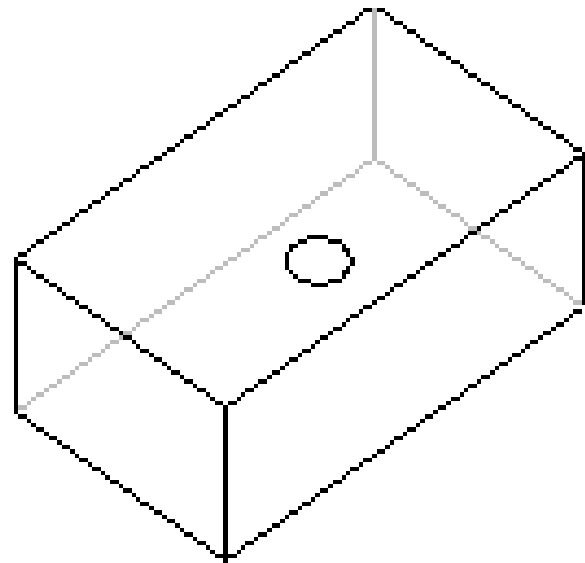
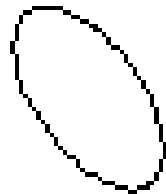
# 2D-3D operacije

- Extrude
- Translacijom 2D konture dobija se prizmatični oblik
- Translacijom se može dodavati ili oduzimati materijal



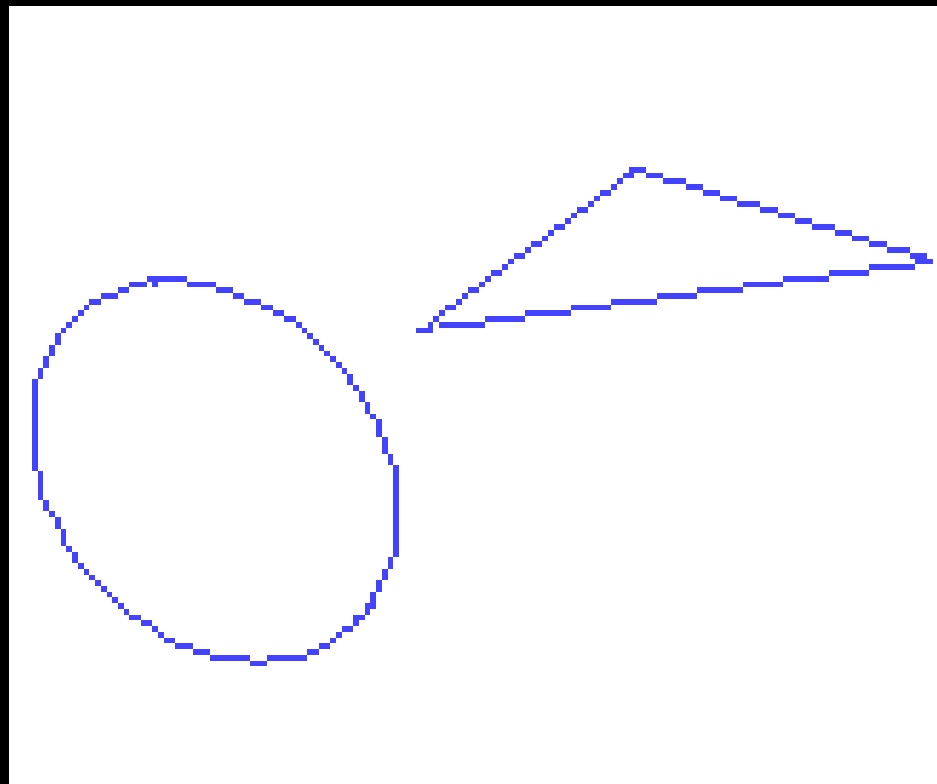
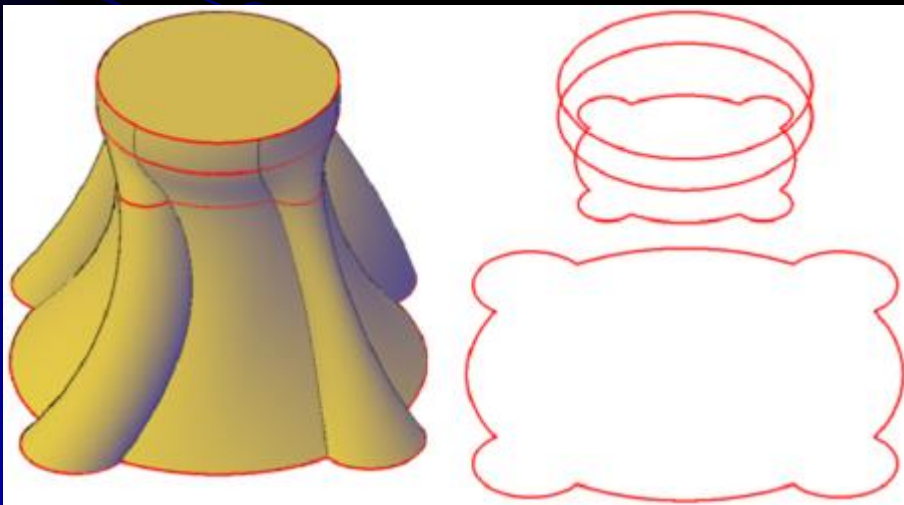
# 2D-3D operacije

- Revolve
- Rotacija 2D konture oko ose



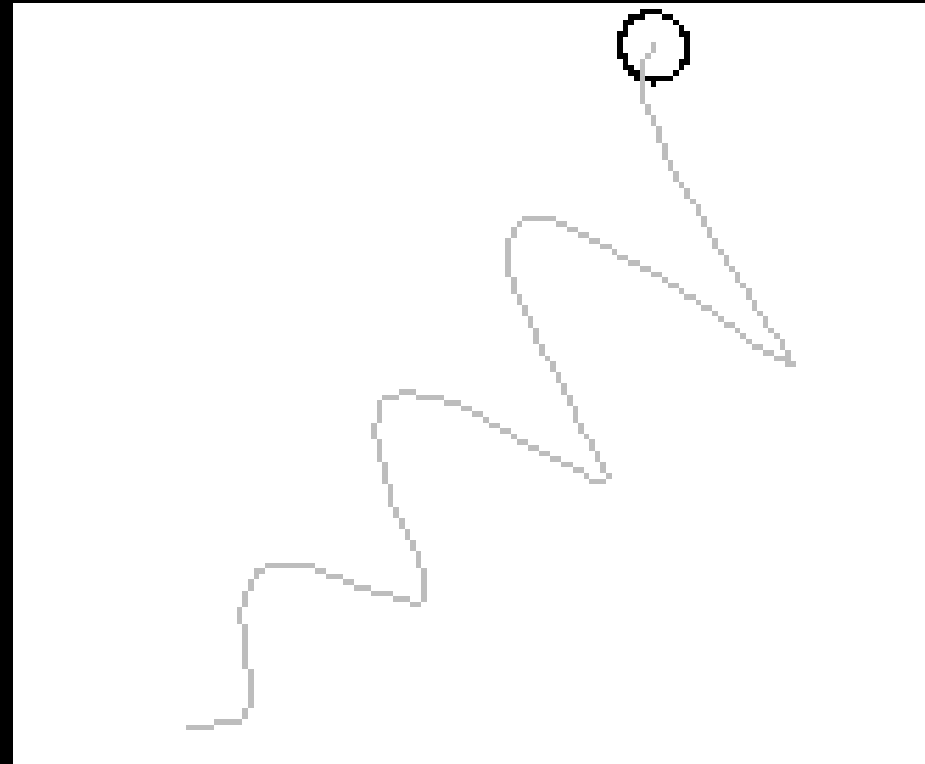
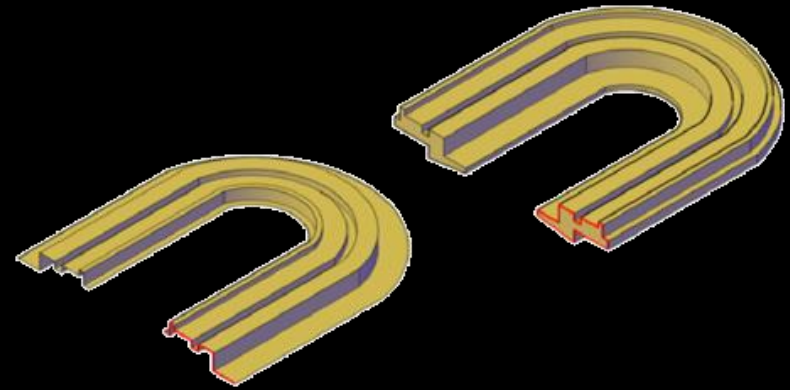
# 2D-3D operacije

- Loft
- Transformacija jedne u drugu 2D konturu; trag koji se pri tome ostavlja predstavlja 3D tijelo



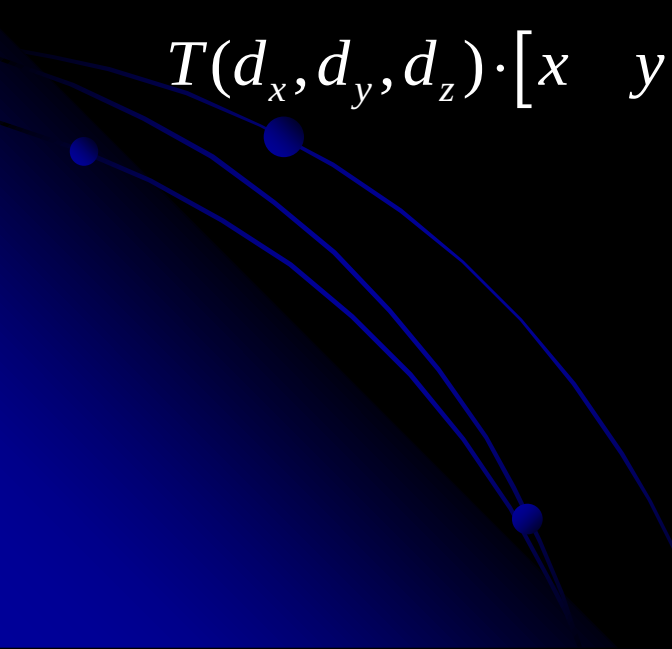
# 2D-3D operacije

- Sweep
- 2D kontura se kreće po krivoj liniji, tako da jedna tačka konture prati krivulju, a ravan konture je uvijek okomita na krivulju.



# 3D translacija

$$T(d_x, d_y, d_z) = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T(d_x, d_y, d_z) \cdot \begin{bmatrix} x & y & z & 1 \end{bmatrix}^T = \begin{bmatrix} x + d_x & y + d_y & z + d_z & 1 \end{bmatrix}^T$$


# 3D rotacija

- Matrice kojim se množi matrica koordinata tačke za rotaciju oko ose x, y ili z:

$$R_z(\varphi) = \begin{bmatrix} \cos \varphi & -\sin \varphi & 0 & 0 \\ \sin \varphi & \cos \varphi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x(\varphi) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \varphi & -\sin \varphi & 0 \\ 0 & \sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\varphi) = \begin{bmatrix} \cos \varphi & 0 & \sin \varphi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \varphi & 0 & \cos \varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# 3D skaliranje

$$S(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S(s_x, s_y, s_z) \cdot \begin{bmatrix} x & y & z & 1 \end{bmatrix}^T = \begin{bmatrix} s_x \cdot x & s_y \cdot y & s_z \cdot z & 1 \end{bmatrix}^T$$
